Geometry Session #4 (Day 3) Pi Day Tresure Hunt Activities!

Today is a special day in our math class! We're going to work our way through a maze of clues to a treasure (which is a fun, no-hands pie eating contest, and kids build and eat the pie all by themselves!) at the end. But first, we'll do our math lesson! To participate fully in this set of activities, you'll need a few materials:

<u>Materials</u>

- string, yarn or similar
- scissors
- masking tape
- measuring tape (up to 10')
- ruler for measuring inches and cm
- circular objects (dish, cup, ball…)
- toothpicks (one box)
- six-sided die (optional)
- coins for markers and tokens
- calculator
- pencil
- Pies for no-hands pie eating contest for the treasure. I like to use small aluminum tart pans with a graham cracker on the bottom, then a dollop of jam, finished by whipped cream. Yum!

<u>Here's what to do:</u>

1. Print out this packet pages 1 - 23
2. Gather your materials (it's ok if you don't have everything, just gather what you have)
3. OPTIONAL: If you have a large group: *print out "pi tickets"* from page 23. Students get one ticket for each correct on the answer sheet they turn in, so print out enough!
4. OPTIONAL: Print pages: 25 – 75, which is a banner if you'd like the first 100 digits of pi. I hang this up on a string near the ceiling if you have a group of students, it is amazing how often the students look up at it as they work through their stations!

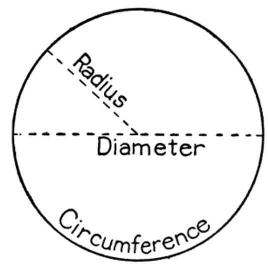
SPECIAL NOTE: If circles are new to your students, please take more time on the first station, which introduces the properties of circles, calculating the ratio of pi first from provided data and then moves into student-led observations and experimentation.

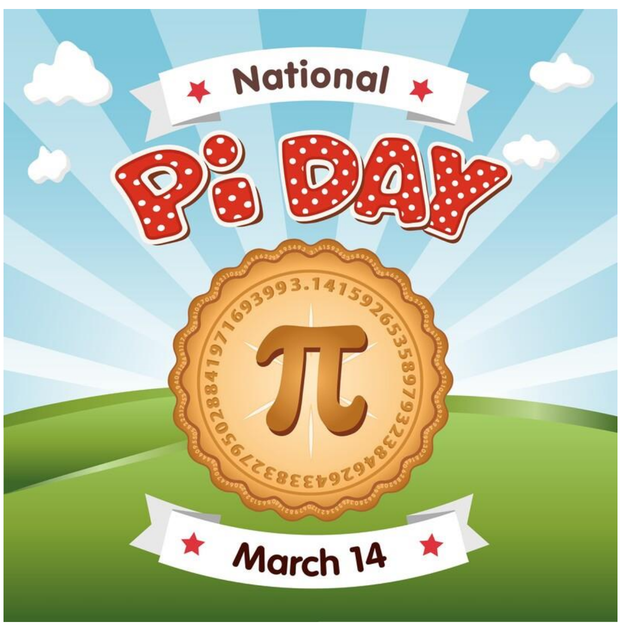
π Pi Day Treasure Hunt Activities

Pi (π) is a number slightly greater than 3, and it shows up in a surprising number of places in math. On March 14 at 1:59pm, folks from all over the world celebrate "Pi Day" with games, activities, and pie-eating contests… and we're going to join in the fun!

But first, let's learn a bit about π (pronounced "PIE"; spelled "pi" or use the Greek letter lower case "π"):

π is the number we get when we divide the circumference of a circle by its diameter. It's always the same number, no matter what size the circle is!





It also shows up in other shapes like spheres, ellipses, cylinders, and cones as well as unusual places like summation series, number theory, probability, bell curves, and the Fibonacci series.

π is an irrational number, which means that the digits never end *and* it doesn't contain any repeating sequences of any length.

Mathematicians can't say with absolute certainty that pi contains every possible finite number sequence—but they strongly suspect that this is the case.

As of March 14, 2022, pi has been calculated to 62.8 trillion decimal places (and this calculation won a Guinness World Record).

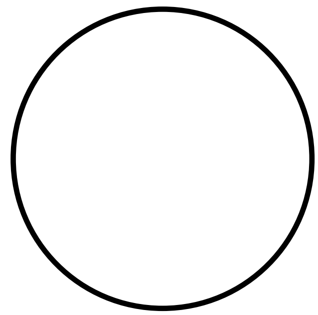
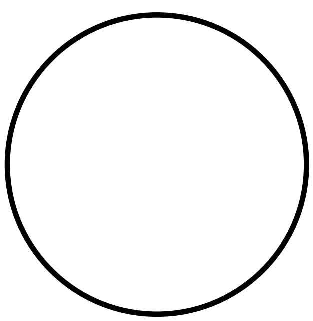
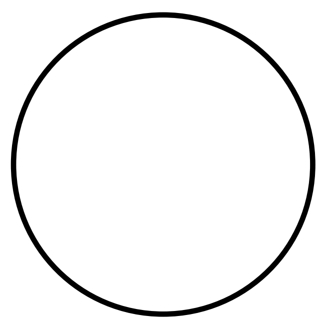
When mathematicians study any sample of this huge number, they find that each digit, 0–9, occurs as often as any other, and that the occurrence of any digit seems unrelated to the preceding digit. This makes pi appear to be statistically random. If this statistical randomness is unending, then pi must contain all finite sequences of digits, including the birth dates of everyone ever born and yet to be born. It also contains every winning lottery number—if only we knew how to identify them!

Here are the first 1,000 digits of π:

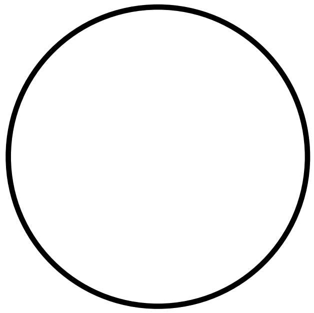
3.1415926535897932384626433832795028841971693993751
058209749445923078164062862089986280348253421170679
821480865132823066470938446095505822317253594081284
811174502841027019385211055596446229489549303819644
288109756659334461284756482337867831652712019091456
485669234603486104543266482133936072602491412737245
870066063155881748815209209628292540917153643678925
903600113305305488204665213841469519415116094330572
703657595919530921861173819326117931051185480744623
799627495673518857527248912279381830119491298336733
624406566430860213949463952247371907021798609437027
705392171762931767523846748184676694051320005681271
452635608277857713427577896091736371787214684409012
249534301465495853710507922796892589235420199561121
290219608640344181598136297747713099605187072113499
999983729780499510597317328160963185950244594553469
083026425223082533446850352619311881710100031378387
528865875332083814206171776691473035982534904287554
687311595628638823537875937519577818577805321712268
0661300192787661119590921642019

<u>Station 1:</u> Discovering π

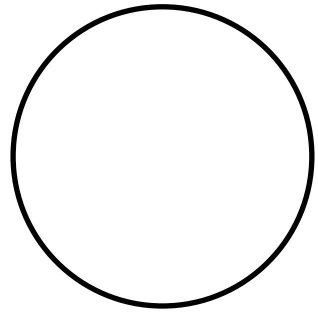
Today we will explore properties of circles and discover the mystery of pi!

A **CIRCLE** is a round shape without corners or line segments.	A **DIAMETER** is any straight line passing through the center of the circle with endpoints *on* the circle. 
The **RADIUS** is measured from the center of a circle to any point on the circle. The radius is half of the diameter of the circle. 	The **CIRCUMFERENCE** is the distance measured *around* the entire circle. 

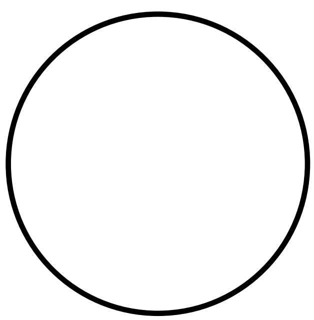
Find the radius of this circle:



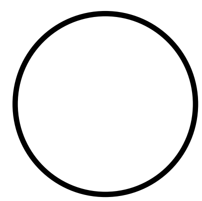
Find the circumference of this circle:



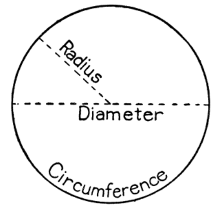
Find the diameter of this circle:



Find the radius of this circle:



Now you try!



Materials

- Ruler or measuring tape
- Calculator and pencil

Steps:

1. Look around the room for circles. Find one you like that you can easily measure, like a dinner plate or circular lid to a jar. (Coins are too small.)
2. Use centimeters as your units to measure the diameter and the circumference of each circle you find. Record your data on the chart below.
3. To complete the last four columns of the chart, use your circumference and diameter measurements to find the sum, difference, product, and quotient of each set of data.

Name of Object	Circumference (C)	Diameter (d)	C + d	C - d	C x d	C ÷ d

Bonus! Do one more measurement using inches instead of centimeters!

Name of Object	Circumference (C)	Diameter (d)	C + d	C - d	C x d	C ÷ d

Did it matter what size the circle is?____

Did it matter what units you measured in?____

What’s Going On?

You probably noticed that one of the columns above gives you close to the same value for every circle. This value is usually a little more than 3, close to the constant ratio pi. The more carefully you make your measurements, the closer this value comes to pi.

Pi is the constant ratio of any circle’s circumference to its diameter. Pi is an irrational number, which means that it cannot be written as a ratio of two integers, and that its decimal expansion goes on forever and is non-repeating.

If we stop the decimal expansion of pi at a certain place, we get an approximation to the number: the more decimal places we retain, the better the approximation we get.

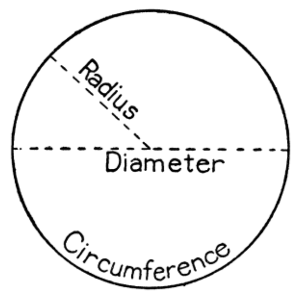
pi = 3.141592653589793...... which is too many numbers to write out each time.

A common approximation is pi ≈ 3.14

<u>Station 2:</u> Cutting π

<u>Materials</u>

- circular object
- string
- scissors
- tape



<u>Steps:</u>

1. Look around the room for circles. Find one you like that you can easily measure, like a dinner plate or circular lid.
2. Carefully wrap string around the circumference of your circular object.
3. Cut the string when it is exactly the same length as the circumference.
4. Now take your “string circumference” and stretch it across the diameter of your circular object. (Watch the directions in class if you chose a 3D object, like a beach ball.)
5. Cut as many “string diameters” from your “string circumference” as you can.
6. How many diameters could you cut?

   Write it here:______________________________

   Compare your data with your friends doing this with you. What do you notice?

What's Going On?

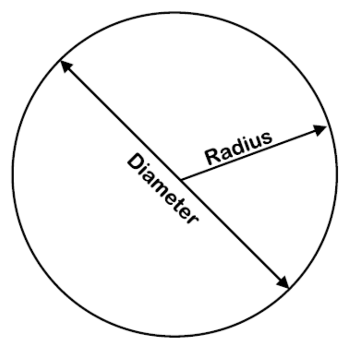
This is a hands-on way to divide a circle's circumference by its diameter. No matter what circle you use, you'll be able to cut 3 complete diameters and have a small bit of string left over. Estimate what fraction of the diameter this small piece could be (about 1/7). You have "cut pi," about 3 and 1/7 pieces of string, by determining how many diameters can be cut from the circumference. Tape the 3 + pieces of string onto paper and explain their significance.

<u>Station 3:</u> π Ball

Is the height of a tennis ball can greater than the circumference of the can?

If you have a can of tennis balls, please use your ruler
to measure the numbers you need.

Otherwise, use the information: Tennis balls have a diameter of 3.25 cm.
Assume the tennis ball touches the top and the bottom of the can (no gaps).



$\pi = 3.14159...$

The Diameter (d)
Any straight line passing through the *center* of the circle with endpoints *on* the circle.

The Circumference (C)
This is the distance measured *around* the entire circle.

$$C = 2\pi r = \pi d$$

The Radius (r)
This is measured *from* the center of a circle to any point *on* the circle.

<u>Station 4:</u> Tossing π

<u>Materials</u>

- toothpicks
- calculator
- drop sheet with lines

<u>Steps:</u>

1. Is the distance between the parallel lines on the sheet the same or greater than the size of the toothpicks?
2. One by one, randomly toss toothpicks onto the lined paper. Keep tossing until you're out of toothpicks.
3. It's time to count:
    a. First, remove any toothpicks that missed the paper or poke out beyond the paper's edge.
    b. Now count up the total number of remaining toothpicks. Also count the number of toothpicks that cross one of your lines.
4. Now figure this out:

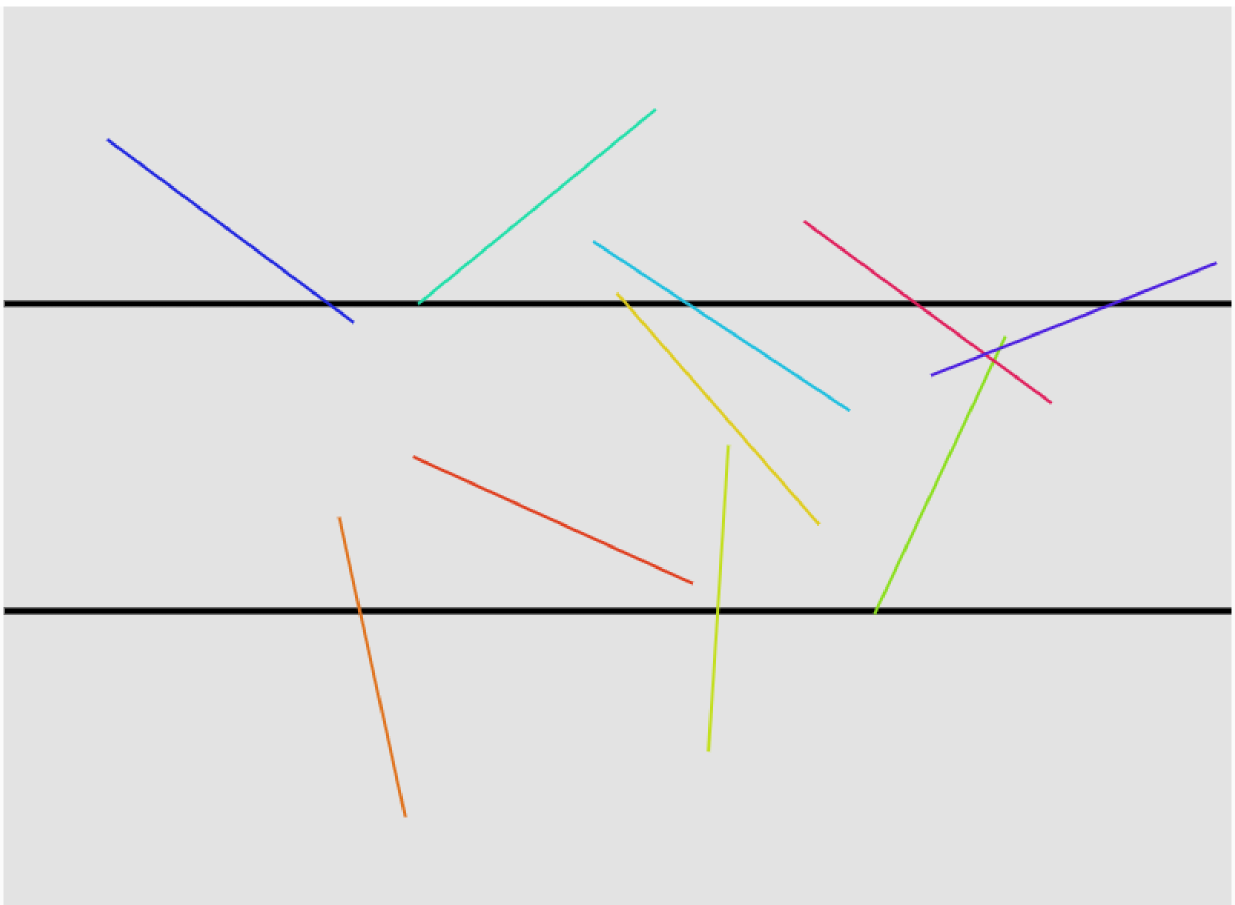
2 × (total number of toothpicks) ÷ (number of line-crossing toothpicks) = ___________________

<u>What's Going On?</u>

This surprising method of calculating pi, known as Buffon's Needles, was first discovered in the late eighteenth century by French naturalist Count Buffon. Buffon was inspired by a then-popular game of chance that involved tossing a coin onto a tiled floor and betting on whether it would land entirely within one of the tiles.

Increasing the number of tosses improves the approximation, but only to a point. This experimental approach to geometric probability is an example of a Monte Carlo method, in which random sampling of a system yields an approximate solution. You can play with a simulation of this activity on this page.

You can see a simulation of Buffon's Needle here: https://mste.illinois.edu/activity/buffon/



Drop 1 | Drop 10 | Drop 100 | Drop 1000 | Drop Amount 10 | Drop

Measurement	Value
Needle Scale	1
Extent = Perimeter / Greatest Vertex Distance	1
Number of Drops	1200
Number of Hits	762
Drops / Hits	1.5748031496062993
π ≈ 2 * Extent * Scale * Drops / Hits	3.1496062992125986

Station 5: π–opoly

First one to get 9 tokens is the winner!

Playing the Game:

1. Roll to see who goes first. Number closes to π goes first.
2. On your turn, roll the die and move your playing piece the number of spaces you rolled.
3. Complete the action on the space. If you answer correctly (either the space action or answer a question card correctly), you get a token.
4. Next person's turn!

Symbols on the Spaces:

For a space marked D (diameter), R (radius), V (volume), and A (area), roll the die for the circumference and then determine the diameter, radius, volume, or area as appropriate.

IMPORTANT! Use a value of 3 for π and help them with the math operations. Correct answer gets a token.

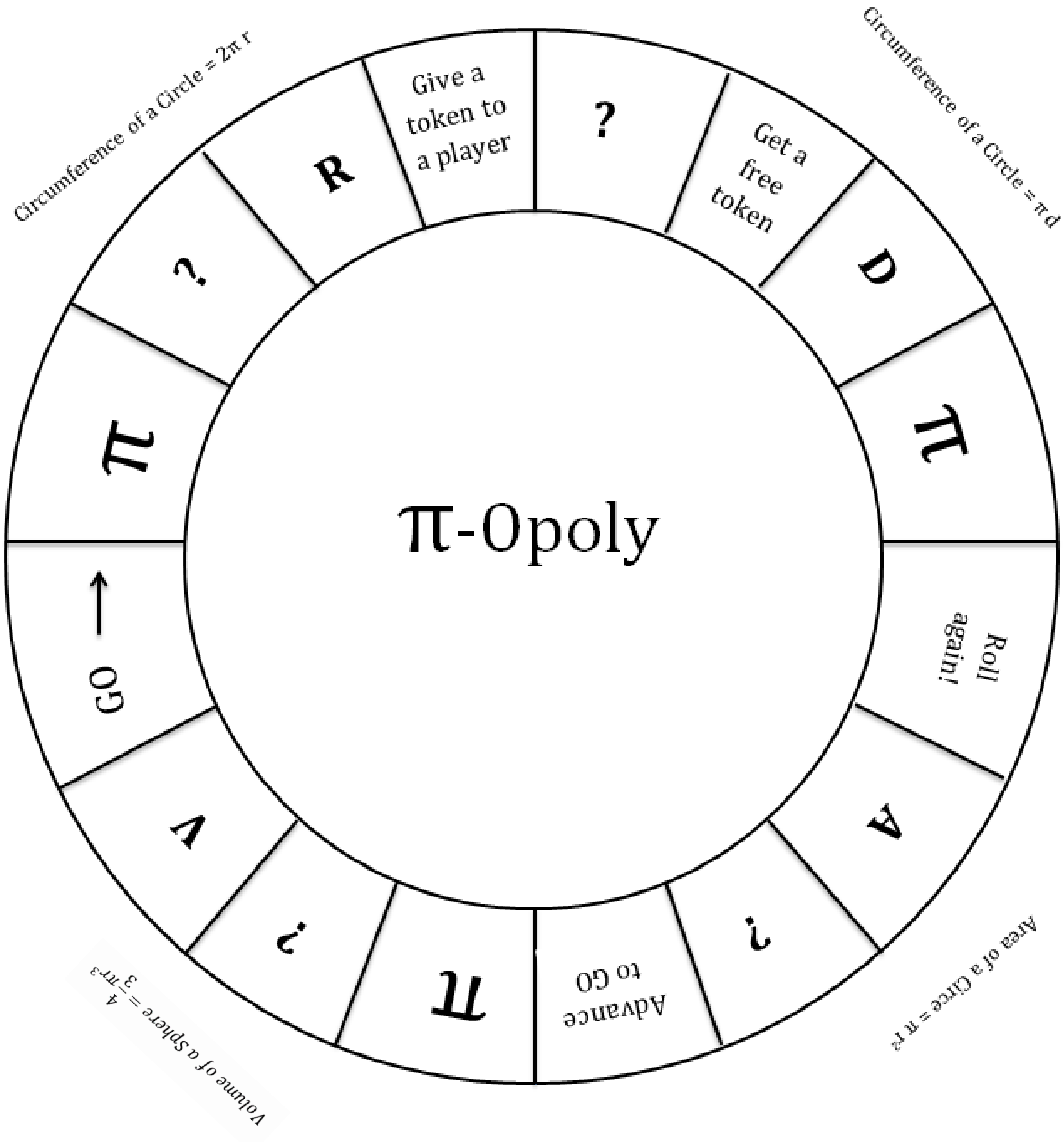
For spaces marked with a "?", pick a question card and answer it. Correct answer gets a token!

For spaces marked with a "π" symbol, recite as many digits of π as you can, and then move forward that many spaces. For example, if you say 3.1415 then move forward 5 spaces. For 3.1415926535, move forward 11 spaces.

Every time you pass GO, take a token!

Play for 8 minutes. How many tokes did you get?_________________________

The area of a circle is 78.5 square inches. Find the circumference. (31.4 inches)	Draw a circle on your paper. Can you split it into 5 equal parts?	Name 4 different kinds of pie.
What is the equation for finding the volume of a ball? ($V = 4/3\ \pi\ r^2$)	How many right angles in a single step of a flight of stairs?	What part of the circle is the circumference? (The line that outlines the circle.)
What is the equation for finding the area of a circle? ($A = \pi\ r^2$)	How many obtuse angles (greater than 90°) on your chair?	Draw the symbol π on paper.
What is the equation for finding the circumference of a ball? ($C = \pi\ d$ or $C = 2\ \pi\ r$)	How many acute angles (less than 90°) on your chair?	What part of the circle is the diameter? (The line drawn across the circle that goes through the center.)
What is the last digit of π? (There isn't one!)	When is π used in the real world?	How is the diameter different from the radius?
What is the circumference of a circle whose diameter is 8 mm? (25.12 mm)	What is the 100th digit of π? (9)	Does π ever end? (No!)
What is the 10th digit of pi? (3)	What is the 1000th digit of π? (9)	Who was the first person to approximate π? (Archimedes)
What language is the symbol π? (Greek)	What fraction approximates π well? (355/113)	What do you get when you divide the circumference of a pumpkin by its diameter? (Pumpkin pie!)
Is π upper or lower case? (Both! π and Π)	What part of a circle is the radius? (From the center to a point on the circle)	What % of sailors are pi rates? (3.14%)
Count by 13's until you get to 100. (13, 26, 39, 52, 65, 78, 91)	What is the volume of a pizza? (Volume = pi * z * z * a where z is the radius and a is the thickness.)	What does this mean? $\sqrt{-1}$ 2^3 Σ π and it was delicious! (I ate some pie and it was delicious!)
Count by 5's until you get to 100. (5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95,100)	What fraction approximates π used by Archimedes? (22/7)	What did *i* say to π, and how did π respond? (*i*: "Be rational!" π: "Get real!")
Count by 3's until you get to 100. (3, 6, 9, 12…)	What part of the circle is the area? (The space contained within the circle.)	What about π is irrational? (Can't be expressed as a fraction of integers.)
Count by 9's until you get to 100. (9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99)	Count by 7's until you get to 100. (7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98)	What is the worst thing about getting hit in the face with π? (It never ends.)


π-0poly
Circumference of a Circle = 2π r
Circumference of a Circle = π d
Area of a Circe = π r²
Volume of a Sphere = 4/3 πr³
Give a token to a player
?
Get a free token
D
π
Roll again!
A
?
Advance to GO
π
?
V
GO ⟶
π
?
R

Station 6: Real π

Which is closest to the real value of π?

(A) $\frac{355}{113}$

(B) $3\,\frac{1}{7}$

(C) $\frac{2^9}{163}$

(D) $\frac{512}{163}$

(E) $\frac{22}{7}$

(F) $\frac{22}{17} + \frac{37}{47} + \frac{88}{83}$

(G) $3\,\frac{10}{71}$

3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172…

Station 7: Searching π

Materials:

- Internet access http://www.angio.net/pi/piquery

Steps:

1. Pick a number sequence that's special to you (your birth date, zip code, favorite number over two digits, your telephone number…)
2. Go to the Pi-Search Page on the computer and type your sequence in the search box at the top of the page.

   This web site will search the first 200 million digits of pi in a fraction of a second. If it finds your sequence, it will tell you at what position in pi your sequence begins and will display your sequence along with surrounding digits.

   No result? Try another sequence. The shorter the sequence, the better the odds of finding it.

3. Now find how many times your zip code occurs in the first 200 million digits of pi. Write this number here: ____________________

What's Going On?

Pi is an irrational number, which means that its digits never end and that it doesn't contain repeating sequences of any length. If Pi-Search didn't find your sequence of numbers, that's probably because the sequence occurs somewhere past the first 200 million digits. Note the qualification "probably": Mathematicians can't say with absolute certainty that pi contains every possible finite number sequence—but they strongly suspect that this is the case.

As of 2011, pi has been calculated to 10 trillion decimal places. When mathematicians study any sample of this huge number, they find that each digit, 0–9, occurs as often as any other, and that the occurrence of any digit seems unrelated to the preceding digit. This makes pi appear to be statistically random. If this statistical randomness is unending, then pi must contain all finite sequences of digits, including the birth dates of everyone ever born and yet to be born. It would also contain every winning lottery number—too bad we don't know how to identify them.

<u>Station 8:</u> Musical π

Arthur Benjamin is a Professor of Mathematics and came up with a funny song about p to the tune of "American Pie" (*Bye, bye Miss American Pie…Drove my Chevy to the levee but the levee was dry…And them good ole boys were drinking whiskey and rye…Singin' this'll be the day that I die…* remember that song now?)

You can watch Prof. Benjamin here: https://www.youtube.com/watch?v=df5zufcPNlY

and then sing along here:

A long long time ago
I can still remember how my math class used to make me snore
Cause every number we would meet
Would terminate or just repeat
But maybe there were numbers that did more

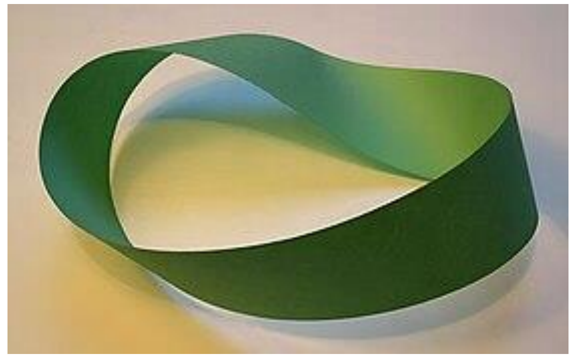
But then my teacher said "I dare ya
To try to find the circle's area"
Despite my every action, I couldn't find a fraction
I can't remember if I cried the more I tried or circumscribed
But something touched me deep inside
The day I learnt of pi
OH

Pi, pi, mathematical pi,
Twice eleven over seven is a mighty fine try
A good old fraction you may hope to supply
But the decimal expansion won't die
decimal expansion won't die

Pi, pi, mathematical pi,
3 point 1 4 1 5 9 2 6 5 3 5 8 9
A good old fraction you may hope to define
But the decimal expansion won't die

What's the last letter of the last word that Prof. Benjamin says in the video?________________

<u>Station 9:</u> Möbius π



A Möbius strip is a surface that only has one surface. We're going to make one in class by cutting off the bottom of this paper (cut along the dashed line), then taking it together like a ring. Before you tape it together, give one of the ends a half-twist and *then* tape it.

This shape was discovered in 1858 by two different mathematicians, but then it was noticed to appear in Roman mosaics from the third century. Another cool property is that there is no "clockwise" or "counterclockwise" with this shape.

Once you have yours made, trace your finger along the surface. What do you notice when you do this? (Hint: try tracing the line with a pencil so you can keep track which side you're on.)

____

In 1977, mathematicians had this idea about the smallest Möbius strip you could possibly make. They tried all different sizes of paper: long and skinny, short and wide… and everything in between. Then they made a *really* short one. It was so stumpy that it flattened out into an equilateral triangle. (Hint: you can shorten the one you created by untapping it and slowly pulling one end to shorten it.)

One thing they did notice was that the ratio of that smallest Möbius strip had a ratio of $\pi \div 2$, which means that the ratio between the length and width of the paper must be greater than pi divided by 1 (~1.57)

Make the smallest Möbius strip and then measure the length and width of your paper:

Length:______________________________

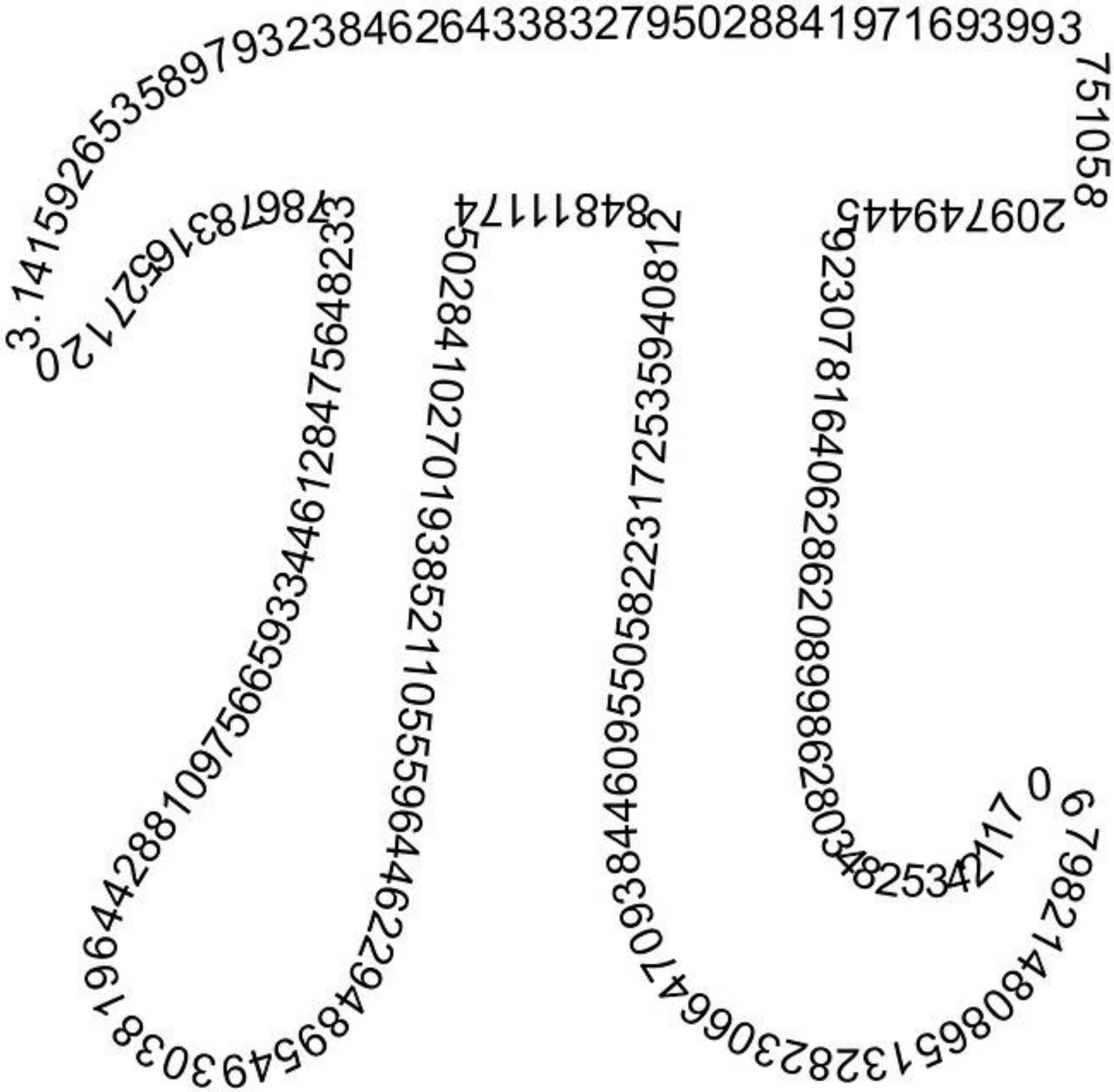
$$\frac{Length}{Width} = \boxed{}$$

Width:______________________________

<u>Station 10:</u> Digital π

What is the 100th DECIMAL digit of pi?________________________

(HINT: Look near the bottom of STATION 6! *How are those numbers grouped?*)



<u>Finale:</u> Decoding π

πιε εατινγ χοντεντ!

τυρν ιν ψουρ ανσωερσ

το γετα τιχκετσ!

<u>Write the alphabet in order for the decode key:</u>

α β χ δ ε φ γ η ι φ κ λ μ

ν ο π θ ρ σ τ υ ϖ ω ξ ψ ζ

π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
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π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!
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π Day Pie Good for 1 build-your-own pie! No hands when eating!	π Day Pie Good for 1 build-your-own pie! No hands when eating!

π Day Answer Sheet:

Station 1. ____(measuring circles)

Station 2. ____(string cutting)

Station 3. ____(tennis ball can)

Station 4. ____(toothpicks)

Station 5. ____(π-opoly)

Station 6. ____(fractions)

Station 7. ____(searching for patterns)

Station 8. ____(singing pi song)

Station 9. ____ (Möbius π)

Station 10. ____ (100^{th} digit)

3.

14

15

92

65

35

89

79

32

38

46

26

43

38

32

79

50

28

84

19

71

69

39

93

75

10

58

20

97

49

44

59

23

07

81

64

06

28

62

08

99

86

28

03

48

25

34

21

17

06

79