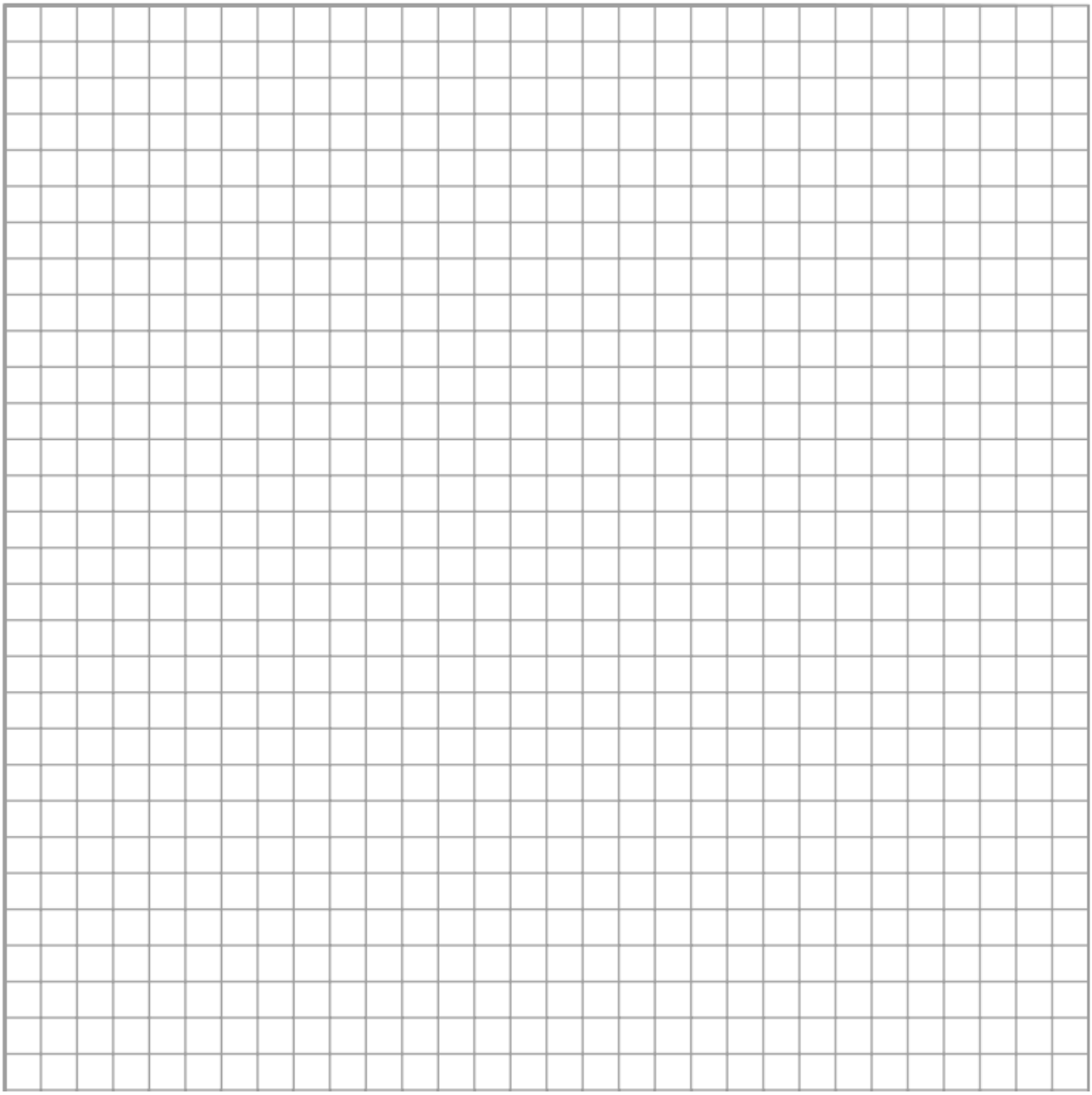
Math Lab #1: Monthly Phone Bill

Plan 1: $10 per month plus 15¢ per minute

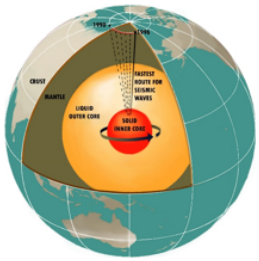
Plan 2: @20 per month plus 10¢ per minute.

How would you decide which is the better plan? What do you have to know in order to make your decision? (Hint: at what point do both plans cost the same?) Graph each equation and find a solution.



Math Lab #2:

Digging through the Earth



$T = 0.013d + 12$

T = Temperature in ^{o}C
d = depth in meters

1. Graph the function
2. What is the slope and y-intercept?
3. How far below the surface will the temperature be 140 ^{o}C ?
4. The deepest mine on Earth is a 3,581 meter gold mine in South Africa. Do they need special cooling equipment for people at this depth?

Math Lab #3:

Supernova and Shock Waves

The expanding supernova shell Cass-A as seen by the Chandra X-ray Observatory (Courtesy: NASA/Chandra)

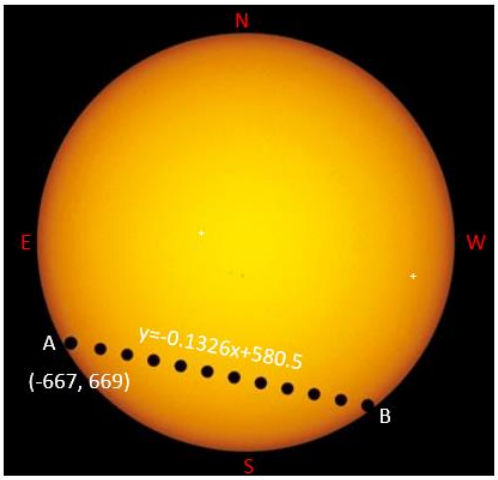
"x" is the ratio of the gas density ahead of the shock wave to the density behind the shock wave.

1. Find the roots!
2. Draw the graph.

$$y = 0.67x^2 + 6x - 2.66$$

Math Lab #4:

Transit of Venus



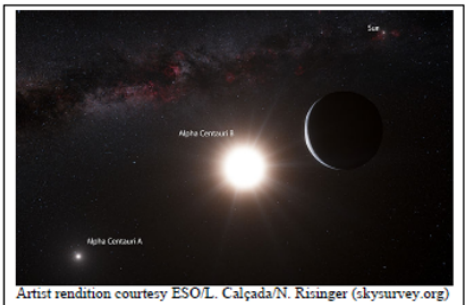
Diameter of the sun = 1890 seconds of arc

1. Write an equation for the circle
2. Find the coordinates of point B
3. If Venus moves at 200 arcsec/hour, how long will the transit take?

$$r^2 = (x-h)^2+(y-k)^2$$

Math Lab #5:

Discovering an Exoplanet



Artist rendition courtesy ESO/L. Calçada/N. Risinger (skysurvey.org)

Time (hours)	Speed (cm/sec)	Time (hours)	Speed (cm/sec)
6	170	48	50
10	150	56	70
21	110	71	130
33	60	83	170

$$T^2 = D^3$$

Alpha Centauri is a binary star system with two stars, A & B. Astronomers measured a wobble in Cen B's orbit speed, which would only be expected if another object is orbiting it.

Let's find that object!

1. Graph the speed data.
2. Draw a smooth curve through the data and estimate the orbit period.
3. How far is this new planet from Alpha Centauri B?

Math Lab #6:

Speed of Sound

Spectacular photo of a fighter jet breaking the 'sound barrier' at a speed of 741 mph or 331 meters/sec (Courtesy Ensign John Gay).

Can you find the speed of sound for the following two days?

1. The coldest day in Antarctica on July 21, 1983 has a recorded temperature of -128°F.
2. The hottest day on Earth was in El Azizia, Libya reaching +80°C.
3. BONUS! What is the speed of sound for an average day of 70°F?

$V = 400T$

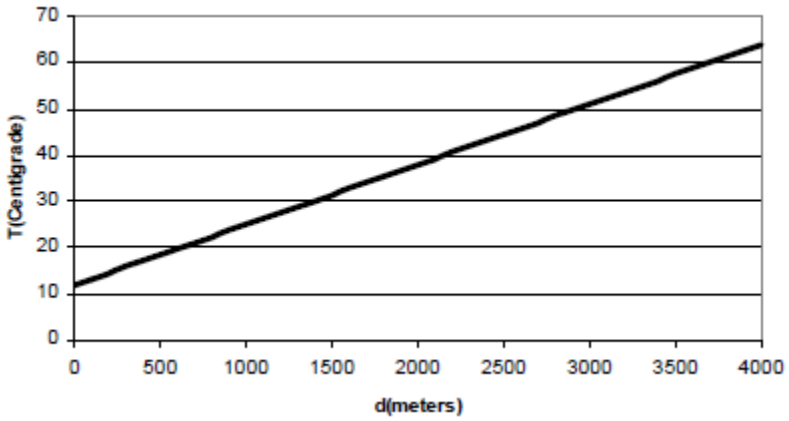
$T = \left(\frac{5}{9}\right)(F - 32)$ $\quad T = C + 273$

ANSWER KEY

Lab #1: Monthly Phone Plan:

If you use over 200 minutes per month of phone time, you want the second plan. But if you are using less than 200 minutes per month then you'll want the first plan. If you're using exactly 200 minutes (not that you'd ever know in advance) then it doesn't matter.

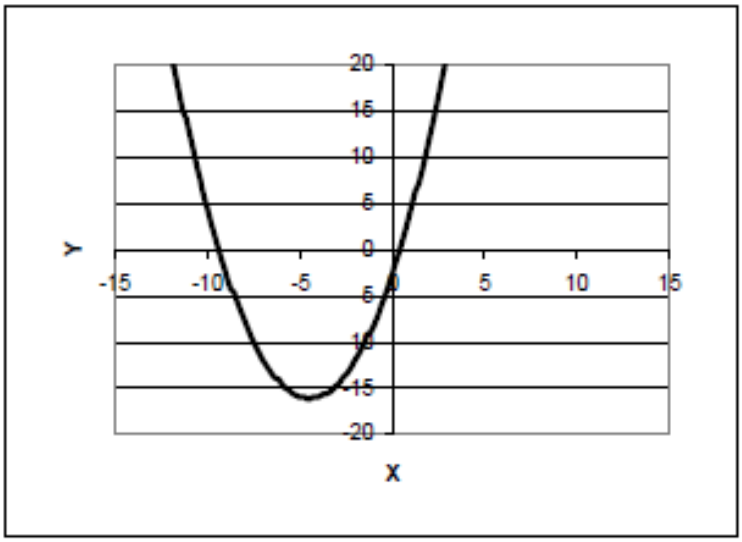
Lab #2: Digging through the Earth



2. Slope = 0.013 meters, y-intercept = 12°C
3. 9.85 km
4. 58 °C, which is 136 °F, which requires special equipment!

Lab #3: Supernova Shock Waves

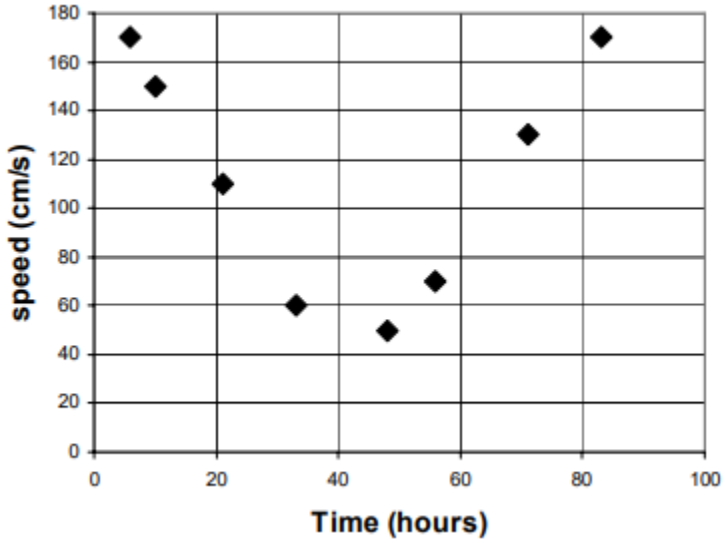
1. Roots: 0.43 & -9.38



Lab #4: Transit of Venus

1. $x^2 + y^2 = 893{,}025$
2. B: (819, 472)
3. 7.5 hours

Lab #5: Discovering an Exoplanet



2. 3.2 days or 0.00877 years
3. 6.4 million km

Lab #6: Speed of Sound

1. 271 m/s
2. 375 m/s

5.3.1

Speed of Sound

$V^2 = 400T$ $\quad$ V = speed (m/s), T = temp (K)

To convert °F to °C: $T(°C) = \frac{5}{9}(F - 32)$

To convert °C to K: $T(K) = T(°C) + 273$

1. Coldest day: $F = -128°F$ $\quad V = ?$

→ need T in K first to use $[V^2 = 400T]$ so:

$T(°C) = \frac{5}{9}(-128 - 32) = -88.89°C$

$T(K) = 273 - 88.89 = \underline{184K}$

$\rightarrow V^2 = 400(184) = 73,640$

$V = \sqrt{73,640} \Rightarrow V = \underline{271 \text{ m/s}}$ (607 mph)

2. Hottest day: $T = 80°C \rightarrow T = 80 + 273 = \underline{353K}$

$\rightarrow V^2 = 400(353) \Rightarrow V = \underline{375 \text{ m/s}}$ (840 mph)

2.3.2.

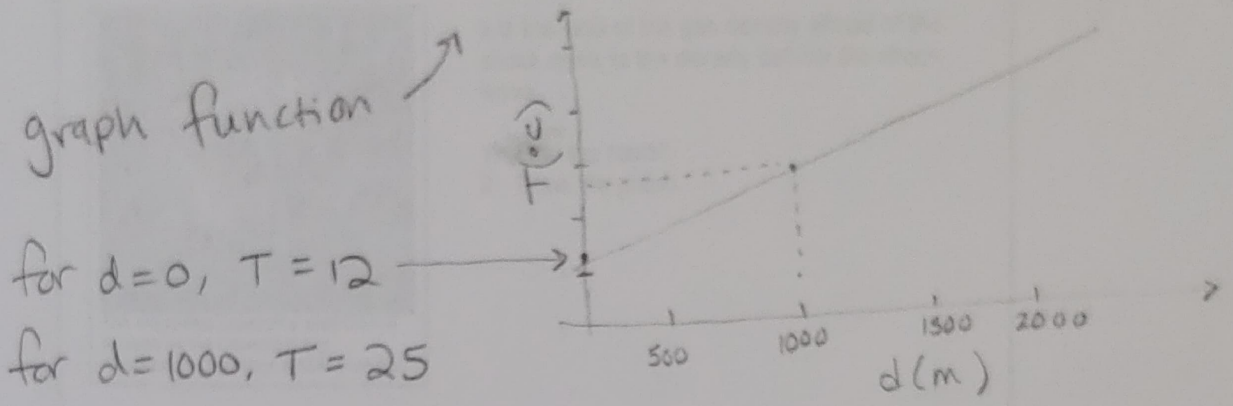
$y = mx + b$ ← y-intercept
↑ slope

Digging thru the Earth

$T = 0.013d + 12$

d = depth (m)
T = temp (°C)

1. graph function



for $d = 0$, $T = 12$
for $d = 1000$, $T = 25$

2. slope = $\frac{\text{rise}}{\text{run}} = \frac{25-12}{1000-0} = 0.013$

3. y-intercept ⇒ $d = 0$ gives $T = 12$

4. for what d is $T = 140°C$? (water boils at 100°C!)

$T = 0.013d + 12$

$(T - 12) = 0.013d \Rightarrow d = \left(\frac{T-12}{0.013}\right)$

$d = \frac{(140-12)}{0.013} = 9850m\ (9.85km)$

5. Deepest mine is at $d = 3581m$. $T = ?$

$T = 0.013d + 12 = 0.013(3581) + 12$

$T = 58°C$ (136°F) → need special cooling equip!

5.6.1.

Supernova + Shock waves

$y = 0.67x^2 + 6x - 2.66$

x = ratio of gas density $\left(\frac{\text{in front}}{\text{behind}}\right)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 0.67$
$b = 6$
$c = -2.66$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(0.67)(-2.66)}}{2(0.67)}$$

$$x = \frac{-6 \pm 6.567}{1.34} \Rightarrow x = \frac{-6 + 6.6567}{1.34} = \underline{0.43}$$

$$x = \frac{-6 - 6.6567}{1.34} = \underline{-9.38}$$

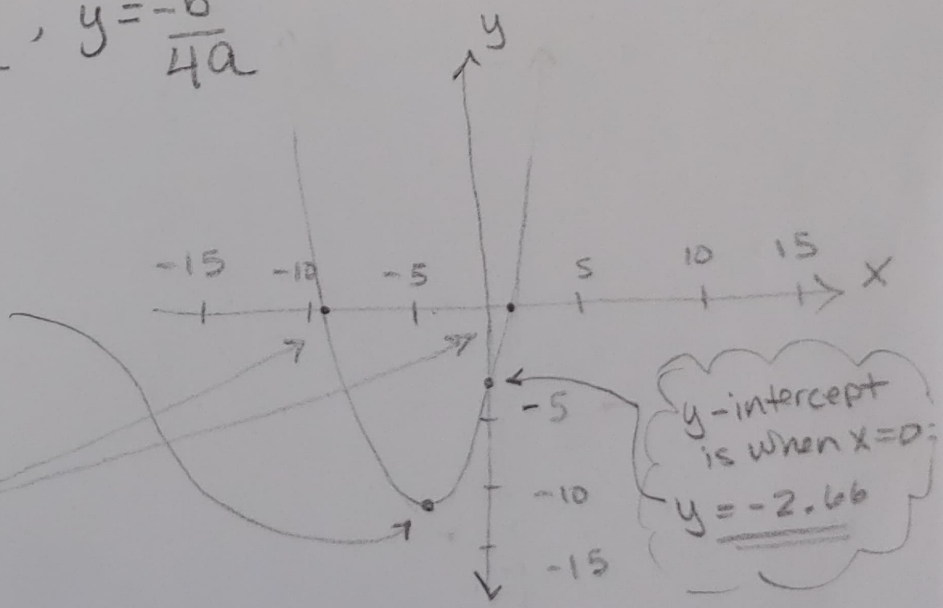
Roots ↑

Note → this number not valid → can't have a negative distance!

Graph: vertex $\Rightarrow x = \frac{-b}{2a}$, $y = \frac{-b^2}{4a}$

$x = \frac{-6}{4(0.67)} = -4.47$

$y = \frac{-(6)^2}{4(0.67)} = -13.4$



x-intercepts = roots!

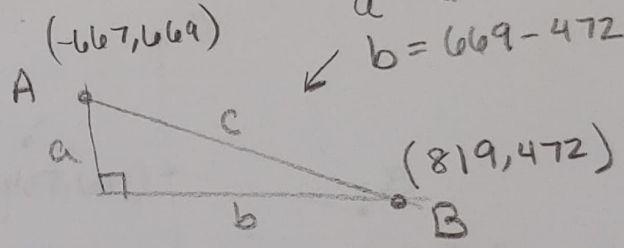
Transit of Venus (cont'd)

$$X = \frac{-(-153.95) \pm \sqrt{(-153.95^2)^2 - 4(1.0176)(-556,045)}}{2(1.0176)}$$

$x = 76 \pm 743 \Rightarrow X_1 = 819 \quad X_2 = -667$ ← we already know this!

$$y_2 = -0.1326(819) + 580.5 = 472.0$$

So pt B $\Rightarrow (+819, +472)$



→ Speed = 200 arcsec/hr

use Pythagorean thm: $a^2 + b^2 = c^2$

$$\left[c^2 = (819+667)^2 + (472-669)^2\right]^{1/2}$$

$c = 1499$ arcsec = distance

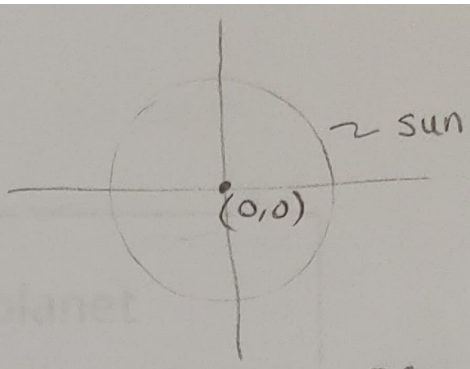
$$\text{Speed} = \frac{\text{distance}}{\text{time}} \Rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$$

1499" / 200"/hr

time = 7.5 hours

10.3.1

Transit of Venus



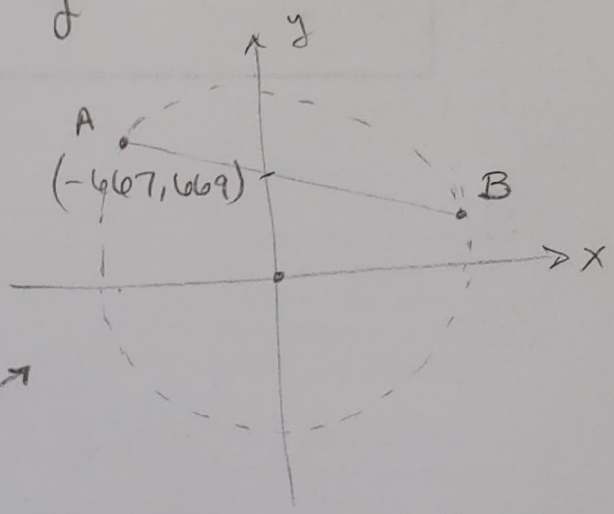
$r = (x-h)^2 + (y-k)^2$

$d_{sun} = 1890$ arcsec $\Rightarrow r_{sun} = \frac{1}{2} d_{sun} = \frac{1890}{2} = 945''$

put (0,0) at center: $(945)^2 = x^2 + y^2$

$$\underline{x^2 + y^2 = 893,025}$$

for (-667, 669) along $y = -0.1326x + 580.5$
find B.



where do circle + line intersect? that will give A + B

$x^2 + y^2 = 893,025$

$y = -0.1326x + 580.5$

$x^2 + (-0.1326x + 580.5)^2 = 893.025$

Simplifies to: $1.0176x^2 - 153.95x - 556,045 = 0$

(use quadratic formula to find roots): $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

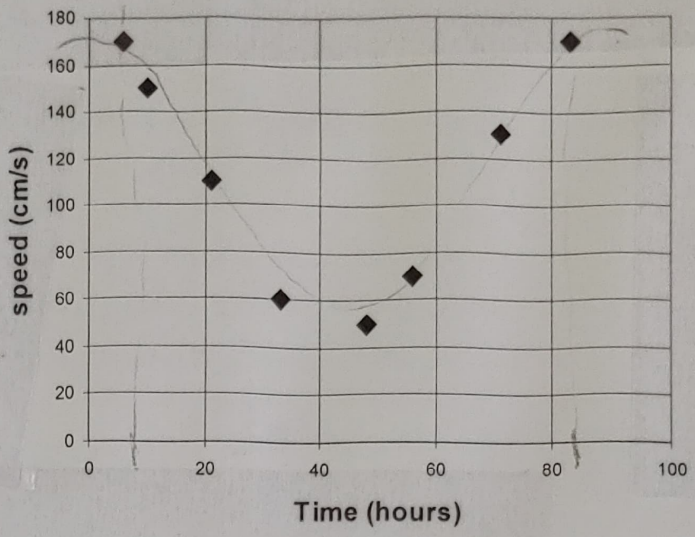
$a = 1.0176$
$b = -153.95$
$c = -556,045$

next page!

#17

Discovering an Exoplanet

graph data →



estimate for speed curve orbit period:

~77 hours (3.2 days)

Kepler's 3rd Law: $T^2 = D^3$

T = years (orbit period)

D = AU (dist to star)

$$T = 3.2 \text{ days} \left(\frac{1 \text{ year}}{365 \text{ days}} \right) = 0.00877 \text{ years}$$

$$D^3 = T^2 = (0.00877)^2 = 7.69E-5$$

$$D = \sqrt[3]{7.69E-5} = \underline{0.043 \text{ AU}}$$

$$D = 0.043 \text{AU} \left(\frac{150E6 \text{ km}}{1 \text{AU}} \right) = \underline{6.4E6 \text{ km}}$$