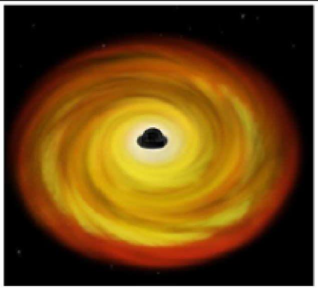
Black Hole Time Travel



An artistic rendition of matter flowing into a black hole at the center of an accretion disk.
(Courtesy M.Weiss NASA/Chandra)

$$T = \frac{t}{\sqrt{1 - \frac{2.8}{r}}}$$

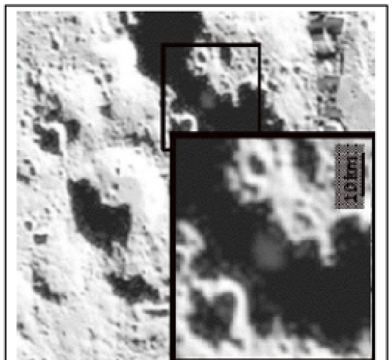
Problem 1 – The Observer knows that the Traveler's clock is ticking once every second so that t = 1.0. Find the equation that gives the distance of the Traveler (r) from the center of the black hole.

Problem 2 – The Observer watches as the Traveler's clock ticks slower and slower.

If the Observer measures the ticks at the intervals of T= 5 seconds, 20 seconds and 60 seconds, how close to the event horizon of the black hole is the Traveler?

The event horizon is 2.8 km from the center of the black hole.

Meteor Impact and Flying Debris



LCROSS debris plume seen in the dark Shadow of a lunar crater. (NASA/LRO)

H = height of particle ejected (meters)
x = distance particle is ejected (meters)
g = acceleration of gravity on moon (2 m/s^2)

1. Factor the equation to find roots.
2. How fast did the debris go?
3. How high did the debris go?

$$H = x - \frac{g}{2V^2}x^2$$

Comet Trails

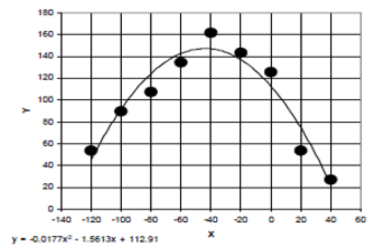
T	W
-120	54
-100	90
-80	108
-60	135
-40	161
-20	144
0	126
+20	54
+40	27

$$W = \frac{(T + 140)(60 - T)}{60}$$

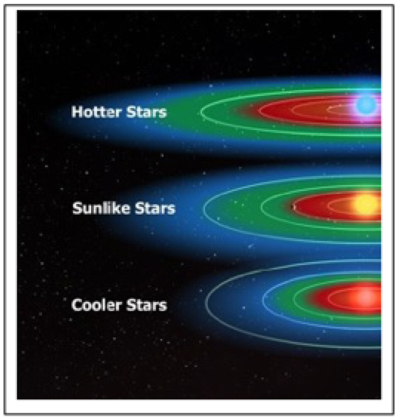
T = number of days since closest approach

W = tons of water produced every minute

1. When does comet stop losing mass?
2. When does it eject the most water per minute?
3. How much water per minute is ejected 130 days before perihelion?



Water on Exoplanets



$$T = 0.6T*\left(\frac{R}{d}\right)^{\frac{1}{2}}$$

In our solar system, what distance from the sun will a planet be warm enough for water to be in a liquid state?

The star Polaris has a temperature of 7200K and a radius 30 times larger than our sun. What is the range that water will be in liquid form?

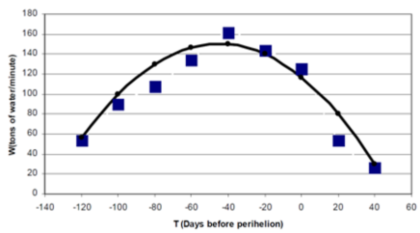
Temperature of our sun = 5770K
Radius of our sun = 700,000 km

<u>Water:</u>
Freezes at 273K
Evaporates at 373K

ANSWER KEY

1. Black Hole Time Travel
    a. $r = \frac{2.8}{1-\frac{1}{T^2}}$
    b. 120 meters to the horizon for 5 seconds
       7 meters to horizon for 20 seconds
       0.8 meters for 60 seconds

2. Meteor Impacts & Craters
    a. $x = \frac{2v^2}{g}$ (final distance from center)
    b. v = 200 m/s
    c. 10 km high

3. Comet Trails
    a. 

    b. -40 days before perihelion
    c. 32 tons per minute

4. Water on Exoplanets
    a. 112 million km
    b. 17AV and 32 AU

7.4.2.

Black Hole Time Travel

$$T = \frac{t}{\sqrt{1 - \frac{2.8}{r}}}$$

t = time on Traveler's clock

T = time interval for Observer

for $t = 1$ find "r" equation

$$\left[T = \frac{t}{\sqrt{1 - \frac{2.8}{r}}}\right]^2 \Rightarrow 1 - \frac{2.8}{r} = \frac{t^2}{T^2}$$

$$1 - \left(\frac{t}{T}\right)^2 = \frac{2.8}{r}$$

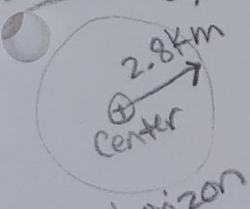
$$r = \left[\frac{2.8}{1 - \left(\frac{t}{T}\right)^2}\right]$$

$t = 1$ so

$$\boxed{r = \frac{2.8}{1 - \frac{1}{T^2}}}$$

for $T = 5$ sec $\Rightarrow r = \frac{2.8}{\left(1 - \frac{1}{5^2}\right)} = 2.92$ Km from center

Black Hole:



2.92 Km − 2.8 Km = 0.12 Km ⇒ 120 m to horizon

7.4.2. Cont'd

for T = 20 sec:

$$r = \frac{2.8}{\left(1 - \frac{1}{(20)^2}\right)} = 2.807 \text{ Km from center}$$

$\underbrace{2.807 - 2.8\text{Km}}_{= 0.007\text{Km}} = 7$ meters to horizon!

for T = 60sec

$$r = \frac{2.8}{\left(1 - \frac{1}{(60)^2}\right)} = 2.8008 \text{ Km from center}$$

2.8008Km - 2.8Km = 0.0008Km = 0.8m to horizon!

5.2.1

Meteorites

$$H = x - \frac{g}{2v^2}x^2$$ average particle path

H = height in meters of average particle

V = particle speed (m/s)

g = acceleration of gravity (m/s^2)

x = distance particle is ejected from impact site

$$H = x - \frac{g}{2v^2}x^2$$ ← Solve for roots

$$H = x\left(1 - \frac{g}{2v^2}x\right)$$

$$H = (x - 0)\left(1 - \frac{g}{2v^2}x\right)$$

$x - 0 = 0$

$\therefore x = 0$ ← starting distance of debris from center of crater

$$1 - \frac{g}{2v^2}x = 0$$

$$1 = \frac{g}{2v^2}x$$

$$\therefore x = \frac{2v^2}{g}$$ ← final distance from center

5.2.1. (Con't) * How fast did the debris go?

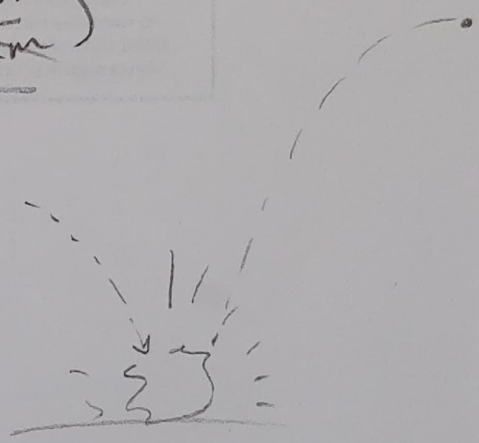
$$\left.\begin{array}{l} g = 2\,m/s^2 \\ X = 40\,Km \end{array}\right\} \quad X = \frac{2v^2}{g}$$

$$\frac{Xg}{2} = v^2$$

$$v = \sqrt{\frac{Xg}{2}}$$

$$V = \sqrt{\frac{(40\,Km)(2\,m/s^2)\left(\frac{1000\,m}{1\,Km}\right)}{2}}$$

$$V = 200\,m/s$$



* How high did the debris go?

$$H = \frac{V^2}{2g} = \frac{(200\,m/s)^2}{2(2\,m/s)} = 10{,}000\,m = 10\,Km$$

(also 6.9.2)

Comet Trails

#49

Comet Tempel-1

$$W = \frac{(T+140)(60-T)}{60}$$

(T = # days since closest approach (perihelion)

W = tons of water ejected per minute

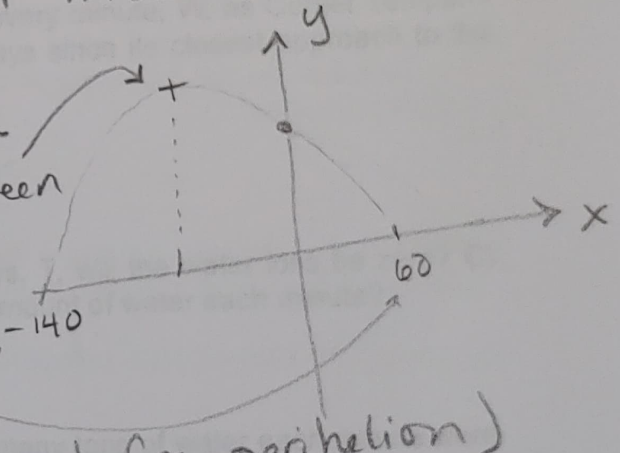
[roots: T = −140 & T = +60]

↳ (use W = 0 to find roots)

☆ when does comet stop losing mass? [W = 0]

☆ when does comet eject most water per min?

↳ at vertex of parabola which is halfway between two roots (intercepts)



↳ $\frac{-140+60}{2} = \underline{\underline{-40}}$ (40 days before perihelion)

☆ How much water per minute is ejected 130 days before Perihelion?

$$W = \frac{(-130+140)(60+130)}{60} = 32 \ \frac{\text{tons}}{\text{min}}$$

7.2.2. Exoplanets + Water

- $T = 0.6T^*\left(\frac{R}{d}\right)^{1/2}$

$R = 700{,}000\,Km$

$T^* = 5770K$

$T = 273\,K$

$$\left(\frac{T}{0.6T^*} = \sqrt{\frac{R}{d}}\right)^2$$

$$\frac{T^2}{(0.6T^*)^2} = \frac{R}{d}$$

- $d = \frac{(0.6T^*)^2 R}{T^2}$

$$d = \frac{[(0.6)(5770k)]^2(700{,}000\,km)}{(273\,K)^2} = \underline{\underline{112.6\text{ million } Km}}$$

— // — Polaris: —— " ——

$T = 273\,K$ to $373\,K$

$T^* = 7200K$

$r = 30x\text{ sun} = 30(700{,}000\,Km) = 21E6\,Km$

- $T = \frac{(0.6)(7200K)\sqrt{700{,}000\,Km(30)}}{\sqrt{d}} = \frac{1.9E7}{\sqrt{d}}$

$d = \left(\frac{1.9E7}{T}\right)^2 \Rightarrow$ T=373 gives $2.6E9\,Km = 17AU$ ← neptune

T=273 gives $4.8E9\,km = 32AU$ ← Pluto