Tidal Forces & Falling into Black Holes

$$a = \frac{2\,G\,M\,d}{R^3}$$

M = mass of the earth = 5.9×10^{27} grams
G = gravitational constant = 6.67×10^{-8} $cm^3 / g \cdot s^2$
R = radius of the Earth = 6.4×10^{8} cm

Typical human height: 200 cm

1. What is the tidal acceleration across the diameter of the Earth?
2. What is the tidal acceleration across a human at a distance of 100 km from a stellar mass black hole?

Mass of black hole = 1.9×10^{33} grams
Radius of black hole = 2.9 km

Radiant Power of a Star

$$T=273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

A = reflectivity (from 0 to 1)
L = luminosity (in multiples of the sun's power)
D = distance between planet and star (in AU)
T is measured in absolute temperature (Kelvin)

1. What is the surface temperature of the Earth if A = 0.4?
2. If the sun's luminosity was increased by 1000, how far would the Earth need to be to maintain the same temperature?

Finding Comets and Asteroids

Close-up of Asteroid Itokawa 50 km across

Kuiper Belt Objects are located between
20 AU and 50 AU from the sun.

Oort Cloud bodies are located more than
50 AU from the sun.

The Asteroid Belt is located between
1.3 and 3.5 AU from the sun.

Short Period Comets orbit between
2 AU and 30 AU.

Long Period Comets orbit greater
than 30 AU.

1. Where should we look to find Short Period Comets in the Kuiper Belt?
2. Where is the Asteroid Belt in km from the sun?

Figuring out the Mass of a Moon

$$F_g = \frac{G M m}{R^2} \qquad F_c = \frac{m V^2}{R}$$

Time of one trip around the moon is 2 hours at a distance of 1,737 km from the moon's center.

Assume that the Apollo-11 went into a circular orbit, and that the gravitational pull from the Earth balances the outward acceleration due to circular motion.

1. What is the equation for finding the mass of the Moon?
2. What is the mass of the Moon in kg?

G = gravitational constant = 6.67×10^{-11} $m^3 / kg \cdot s^2$

ANSWER KEY

Tidal Forces & Falling into Black Holes

1. 1, 3,800 cm/sec^2
2. 51,000,000 cm/sec^2

Radiant Power of a Star

1. 240K
2. 32 AU

Finding Comets and Asteroids

1. 20 AU < D < 30AU
2. 195 million km < D < 525 million km

Finding Mass of the Moon

1. $M = \frac{RV^2}{G}$ simplifies to $M = \frac{4\pi^2 R^3}{GT^2}$
2. M=6.00 x 10^{22} kg

1.2.2.

Black Hole Tidal Forces

$$a = \frac{2GMd}{R^3}$$

M = 5.9E27 g

R = 6.4E8 cm

G = 6.67E-8 $\frac{\text{dynes cm}^2}{g^2}$

1 dyne = 0.00001 N = 1 g cm/s²

1. d = 2R = 1.28E9 cm

$$a = \frac{2(6.67E\text{-}8 \frac{g\,cm\,cm^2}{s^2 g^2})(5.9E27\,g)(1.28E9\,cm)}{(6.4E8\,cm)^3}$$

$a = 3{,}800\ cm/s^2$

2. m = 1.9E33 g

$R = 100\,km \left(\frac{1000\,m}{1\,km}\right)\left(\frac{100\,cm}{1\,m}\right) = 1E7\,cm$

$$a = \frac{2GMd}{R^3}$$

d = 2m = 200 cm

$$a = \frac{2(6.67E\text{-}8 \frac{g\,cm^3}{s^2 g^2})(1.9E33\,g)(200\,cm)}{(1E7\,cm)^3}$$

$a = 51{,}000{,}000\ cm/s^2$ ← this is 52,000 times the earth's gravity so a person would be pulled apart

1.2.7. Radiant Power of a Star

1. Surface Temp

$$T = 273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

A = reflectivity = 0.4

L = luminosity = 1.0

D = distance = 1.0 AU

$$T = 273\left(\frac{(1-0.4)(1.0)}{(1.0)^2}\right)^{1/4}$$

$$T = 273(0.6)^{1/4} = \underline{\underline{240\text{ K}}}$$

2. D = ? T = 240K, L = 1000, A = 0.4

$$T = 273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

$$\frac{T^4}{(273)^4} = \frac{(1-A)L}{D^2}$$

$$D^2 = \frac{(1-A)L}{T^4}(273)^4$$

$$D^2 = \frac{(1-0.4)(1000)}{(240K)^4}(273)^4 \Rightarrow \underline{\underline{D = 32\text{ AU}}}$$

1.6.2

Asteroids & Comets

asteroid belt: 1.3 – 3.5 AU : $1.3\,AU < D < 3.5\,AU$

Kuiper belt: 20 – 50 AU : $20\,AU < D < 50\,AU$

oort cloud: 50+ AU : $D > 50\,AU$

S.P. Comets → orbits 2 AU → 30 AU : $2\,AU < D < 30\,AU$

L.P. Comets → orbits 30^{+} AU : $20\,AU < D < 50\,AU$

1. Where are S.P. Comets in Kuiper Belt?

$$\boxed{20\,AU < D < 30\,AU}$$

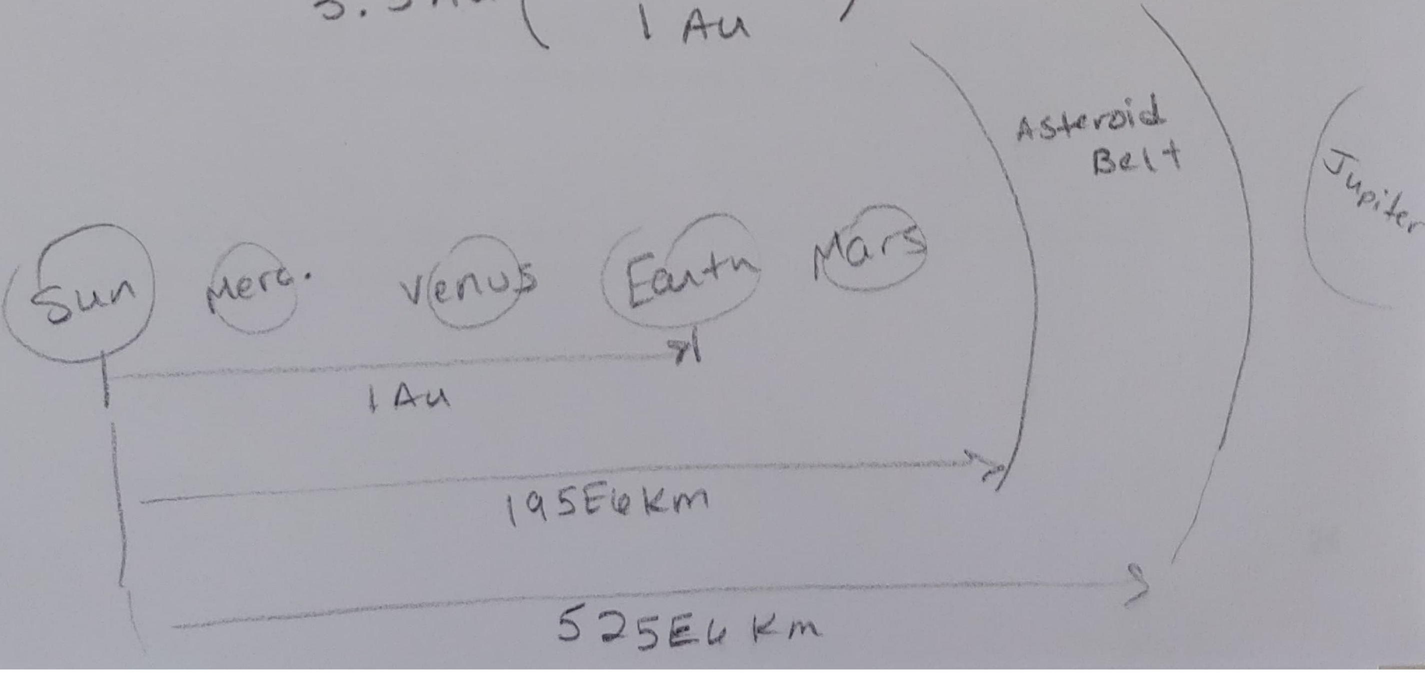
2. where is asteroid belt?

$$1\,AU = 150E6\,Km$$

$$1.3\,AU\left(\frac{150E6\,Km}{1\,AU}\right) = 195E6\,Km$$

$$3.5\,AU\left(\frac{150E6\,Km}{1\,AU}\right) = 525E6\,Km$$

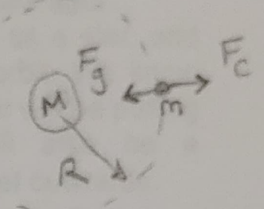
*difference gives width of belt!



1.4.1

Moon Mass

Assume a circular orbit & the inward grav accel by moon (F_g) equals outward centrifugal accel. (F_c)



$$F_g = \frac{GMm}{R^2}$$

$$F_c = \frac{mv^2}{R}$$

$$\frac{GM\cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}}$$

$$\frac{GM}{R} = v^2 \Rightarrow M = \frac{v^2 R}{G}$$

2. $$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi R}{T} \Rightarrow M = \frac{\left(\frac{2\pi R}{T}\right)^2 R}{G}$$

simplify to:

$$M = \frac{4\pi^2 R^3}{GT^2}$$

3. $T = 2\text{hours} = 7200\text{ seconds}$

$R = 1.737E6m$

$$M = \frac{4\pi^2 (1.737E6m)^3}{6.67E{-11}\frac{m^3}{Kg \cdot s^2}(7200s)^2} \Rightarrow M = 6.00E22Kg$$

(published value: 7.4E22Kg)